Sujet 1

Here are two math exercises.

1) **Choose one of them and explain your choice.**
2) **Try to solve one of them –** ***if you can't managed to find an exact solution, explain what you have done-***
3) **Give an overview of the second one.**

EXERCISE 1

Let ABC be a scalene triangle (AC<AB) and $\mathcal{C}$ the circle of centre A and radius AC.

The line (AB) intersects the circle in D and E, with D on [AB]. The line $\mathcal{D}$ is parallel to (CE) and passes through A.

The aim of this exercise is to prove that $\mathcal{D}$ is the angular bisector of the angle $\widehat{BAC}$.

1) Draw a precise figure
2) Use a classic configuration to find what kind of triangle DCE is.
3) What can you deduce about the line $\mathcal{D}$, (DC) and (CE).
4) What can you deduce about the line $\mathcal{D}$ in the triangle ADC?
5) Find out what kind of triangle ADC is.
6) Conclude.

EXERCISE 2

A population of 3000 flies is increasing at a rate of 3% per minute.

How many flies will be alive after 10 minutes?

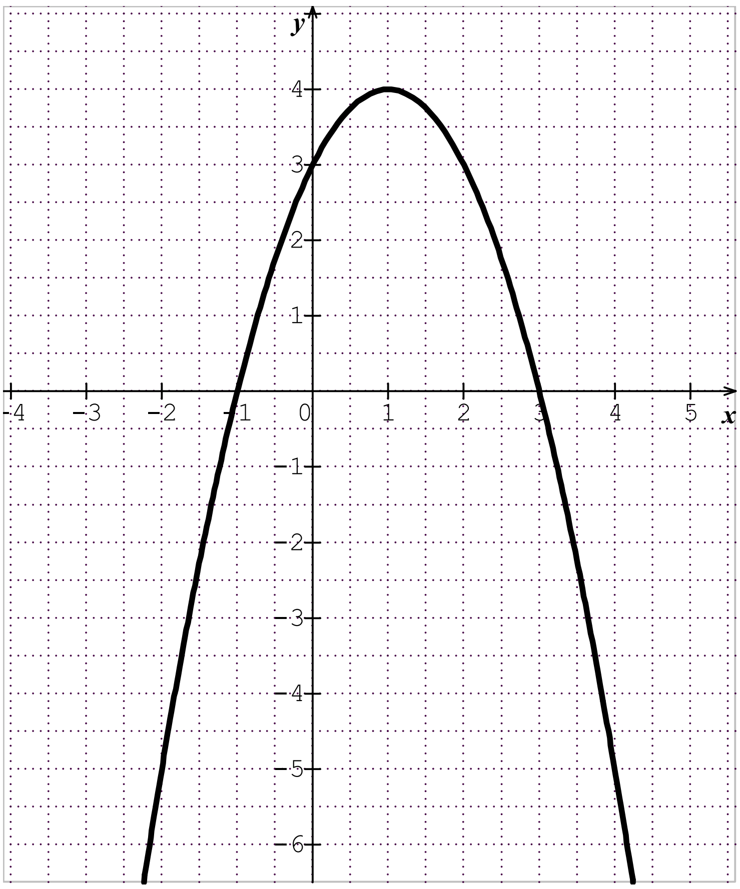
Sujet 2

Here are two math exercises.

1) **Choose one of them and explain your choice.**
2) **Try to solve one of them – *if you can't managed to find an exact solution, explain what you have done-***
3) **Give an overview of the second one.**

EXERCISE 1

The graph of a particular function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, is shown below. Use the graph to find a, b, and c.



EXERCISE 2

A mine worker discovers an ore sample containing 500 mg of radioactive material.

It is discovered that the radioactive material has a half life of 1 day.

Find the amount of radioactive material in the sample at the beginning of the 7th day.

Sujet 3

Here are two math exercises.

1) **Choose one of them and explain your choice.**
2) **Try to solve one of them – *if you can't managed to find an exact solution, explain what you have done-***
3) **Give an overview of the second one.**

EXERCISE 1

An arrow is shot into the air; its height is given by $h(t) = -4.9t^2 + 30t + 2$ meters above the ground.

t is the time in seconds.

1) Determine the maximum height of the arrow and when it occurs.
2) When does the arrow hit the ground?
3) At what time is the arrow 40 meters in the air?

EXERCISE 2

1) Given is an arithmetic sequence $\{a_n\}$ with a first term $a_1 = 15$ and the common difference $d = 8$, find the term a_{101}.
2) For an arithmetic sequence, the terms $a_8 = 21$ and $a_{10} = 25$ are known.
   Find the terms a_0 and a_n.

Sujet 4

Here are two math exercises.

1) **Choose one of them and explain your choice.**
2) **Try to solve one of them – *if you can't managed to find an exact solution, explain what you have done-***
3) **Give an overview of the second one.**

EXERCISE 1

An open box is to be constructed from a piece of cardboard 20 inches by 20 inches by cutting squares of side length x from each corner and folding up the remaining sides.

Round answers to one decimal place.

a) Draw a figure.

b) Express the volume V of the box as a function of x, and find the domain.

c) Graph V (x).

d) Find the maximum volume.

EXERCISE 2

1) Explain what the mathematical induction is?
2) Prove by mathematical induction that for all positive integers n, $1+2+\ldots+n=\dfrac{n(n+1)}{2}$.

Sujet 5

TEXT

Definition

Two events of the same experiment are mutually exclusive if they cannot occur simultaneously

Theorem : Conditional probability

If E_1 and E_2 are two events (not necessarily from the same experiment), then the probability that E_1 will occur given that E_2 has occured is :

$$P(E_1|E_2) = \frac{P(E_1 and\ E_2)}{P(E_2)}$$

If E_1 and E_2 are mutually exclusive, then :

$$P(E_1|E_2) = 0$$

Two events are independent if :

$P(E_1) = P(E_1|E_2)$ and $P(E_2) = P(E_2|E_1)$

EXERCISE

At Roundway petroleum station, 30% of customers buy 4-star petrol, 60% buy unleaded petrol and the remainder buy fuediesel. When a customer buys 4-star petroleum there is 25% chance that she/he will fill the tank. Customers buying unleaded petrol have an 80% chance of not filling the tank. Of those buying diesel, 70% fill their tank.

a) Draw a probability tree diagram to illustrate this situation

b) What is the probability that when a car leaves the petrol station it does not have a full tank ?

c) Given that a car leaving the petroleum station has a full tank, what is the probability that the tank contains unleaded petrol ?

<u>Vocabulary :</u> *there are three sorts of petroleum in England, as in France : petrol, unleaded petrol and diesel*

QUESTIONS

1. Explain, in your words, the theorem. Have you seen it this year ?
2. Give an example using this theorem in a calculation of probability.
3. Explain your way to solve the exercise

Sujet 6

TEXT

Definition

Two events of the same experiment are mutually exclusive if they cannot occur simultaneously

Theorem : Conditional probability

If E_1 and E_2 are two events (not necessarily from the same experiment), then the probability that E_1 will occur given that E_2 has occured is :

$$P(E_1|E_2) = \frac{P(E_1 and\ E_2)}{P(E_2)}$$

If E_1 and E_2 are mutually exclusive, then :

$$P(E_1|E_2) = 0$$

Two events are independent if :

$P(E_1) = P(E_1|E_2)$ and $P(E_2) = P(E_2|E_1)$

EXERCISE

4 people were chosen at random from a group of 8 which comprised 4 husbands and their 4 wives. Find the probability that the sample contained :

1. One person from each married couple (called event A)
2. 2 males and 2 females (called event B)
3. Find P(A|B) (the probability that event B happened if it is known that event A happened)

QUESTIONS

4. Explain, in your words, the theorem. Have you seen it this year ?
5. Give an example using this theorem in a calculation of probability.
6. Explain your way to solve the exercise

Sujet 7

TEXT : ARITHMETIC SEQUENCES

- DEFINITION : an arithmetic sequence is a sequence whose terms go up or down by constant steps.
- An arithmetic sequence has the form : $u_1 = a$ $u_{n+1} = u_n + d$. The number d is called the common difference.
- It is possible to calculate the n-th term of an arithmetic sequence in terms of a and n :

  $U_n = a + (n - 1)d$
- The sum of the arithmetic series formed by adding the terms of an arithmetic sequence is called S. It is possible to find a formula so as to calculate S in terms of a, d and u_n :

  $2S = n(a + u_n)$ or $2S = n(2a + (n - 1)d)$

EXERCISES

1. Find the sum of the first n terms of an arithmetic sequence defined by : a = 100 and d = -3.
2. Find the sum of the arithmetic sequence : 1, 4, 7, 10, 13, …, 1000.
3. An arithmetic sequence has first term a and a common difference 10. The sum of the first n terms of the sequence is 10000. Express a in term of n, and show that the n-th term of the sequence is : $\frac{10000}{n} + 5(n-1)$.

QUESTIONS

Q.1. Read the text.

Q. 2. Explain, in your own words, the meaning of the text. Did you study it during the French math course? the English math course?

Q.3. Explain a way to find both formulas given in the text.

Q.4. Present your way of solving the exercises, if you know it, or, if don't know it, explain the calculus you made.

Sujet 8

TEXT : GEOMETRIC SEQUENCES

- DEFINITION : an geometric sequence is a sequence defined by : $u_1 = a$ and $u_{n+1} = qu_n$, where n is a natural number and q different from 0 and 1. The constant q is called the common ratio.
- It is possible to calculate the n-th term of a geometric sequence in terms of a, q and n : $u_n = a\,q^{n-1}$
- The sum of the geometric series formed by adding the terms of a geometric sequence is called S. It is possible to find a formula so as to calculate S in terms of a, q and u_n : $S = \frac{a(1-q^n)}{1-q}$

EXERCISES

4. A geometric sequence is given by : 3, 6, 12, … . Find the common ratio and the two next terms. Find an expression of the n-th term.

5. Find the common ratio and the first term of the geometric sequence given by :
   The 3rd ter mis 6 and the 7th ter mis 96.

6. a child lives 200 meters from school. He walks 60 meters in the first minute, and in each subsequent minute he walks 75% of the distance he walked in the previous minute. Show that it takes to him between 6 and 7 minutes to get school.

QUESTIONS

Q.1. Read the text.

Q. 2. Explain, with your own words, the meaning of the text. Did you study it during the French math course? the English math course?

Q.3. Explain a way to find both formulas given in the text.

Q.4. Present your way of solving the exercises, if you know it, or, if don't know it, explain the calculus you made.

Sujet 9

TEXT : OPTIMISATION

- x_0 is a point of local maximum of a function f signifies :

  that f'(a) = 0 , that if x < a f'(x) < 0 and that if x > a f'(x) > 0.

- x_0 is a point of local minimum of a function f signifies :

  that f'(a) = 0 , that if x < a f'(x) > 0 and that if x > a f'(x) < 0.

- At a point of local maximum a function has a geater value than at points immediately on either side of it.
- At a point of a local minimum a function has a smaller value than at points immediately on either sides.
- Local maxima and minima are also called turning points. A function may have more than one turning point. The local maxima or minima are not necessarily the greatest or least values of the function.

EXERCISES

1. Calculate the coordinates of the maximum and the minimum points on the curve given by the equation : $y = x^2 (a - x^2)$, where a is a positive constant.
2. A function h is defined by : $h(x) = (ax^2 + b) e^{cx}$. It is known :

   that h(0) = - 4, that h'(0) = 8 and that the function has a minimum value at x = - 1.

   Find the values of a, b and c.

3. A groundsman has a 80 meters of tape with which to mark out a rectangular enclosure against a brick wall. The length of each of the two shortest sides is x meters and that of the longest side is y meters.
   a) Show that the area of of the enclosure, A m^2, is given by :

      $A = 80x - 2x^2$.

   b) Find the maximum area that can be enclosed.

QUESTIONS

Q.1. Read the text.

Q. 2. Explain, with your own words, the meaning of the text. Did you study it during the French math course? the English math course?

Q.3. Present your way of solving the exercises, if you know it, or, if don't know it, explain the calculus you made.